Lab Report 2: Fourier Imaging, Spatial Fourier Transforms

Group: Pranav Reddy, Altug Ozcetinkaya, Christopher Hemsley, Dashiell Franklin, Gavin Kesler, Karl Tizon, Nicholas Shand

Electrical and Computer Engineering Department, University of California, Santa Barbara

1 Executive Summary

The lab explored the principles of Fourier imaging and spatial filtering through three different experiments. Through a 4-f imaging system, the first part explored how aperture and field iris affected image resolution and brightness. These correlations resulted from the Rayleigh criterion. The second part used slit filtering on the Fourier plane, selecting spatial frequency components and seeing its effect on the image. This confirmed the relationship between the gratings and diffraction orders. The third part of the experiment explored how different patterns proved the Babinet's principle and contrasting aperture patterns affect intensity distributions. These experiments show how fourier plane manipulation affects resolution and brightness through controlling and filtering spatial frequencies

2 Introduction

The lab focused on fourier optics through spatial filtering and image processing.

The first objective of the lab is to explore the fourier image. In a 4-f system the lens performs a fourier transform on the object plane, mapping its corresponding spatial frequencies as planar waves. The back of the lens captures the fourtier transform where spatial frequencies of the initial object will appear. When coherent light illuminates the object on the front focal plane of a lens then the fourier image will appear as bright spots, diffraction patterns, on the back focal plane of the lens which is performing the fourier transform. Mathematically, the wave is broken down into its fourier transform pair between the object plane and post fourier transform on the back half of the lens:

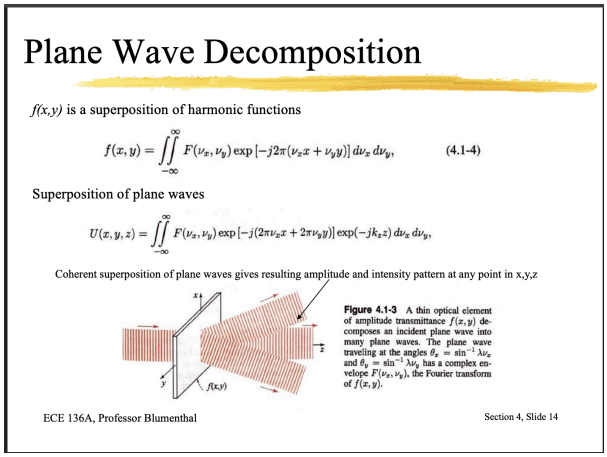


Figure 1: Pupil Imaging Function from Section 4, slide 4

The pupil function governs the systems transfer function, affecting the transmitted spatial frequencies. The aperture takes the frequencies and multiplies it by the pupil function, affecting the resulting diffraction pattern. By varying both the aperture and field iris the fourier image is manipulated, and by observing the fourier plane, these effects can be observed.

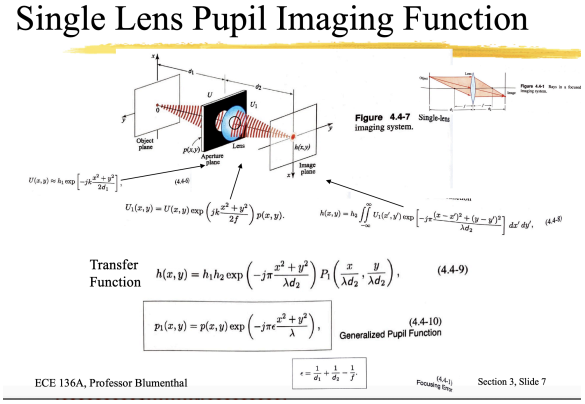


Figure 2: Pupil Imaging Function from Section 3, slide 7

Aperture size and diffraction effects are considered in relation to resolution. From the Rayleigh criterion, two points are resolvable at the maximum of one diffraction pattern. This pattern relates to the first minimum of the other. The objective is to explore the relation of aperture diameter size on resolution and brightness. Aperture size explores the relationship between ray optics and fourier imaging principles where filtered frequencies of lights affect the resolution and brightness of images. The aperture size's diameter is determined by an optical element whose knob controls pinhole size.

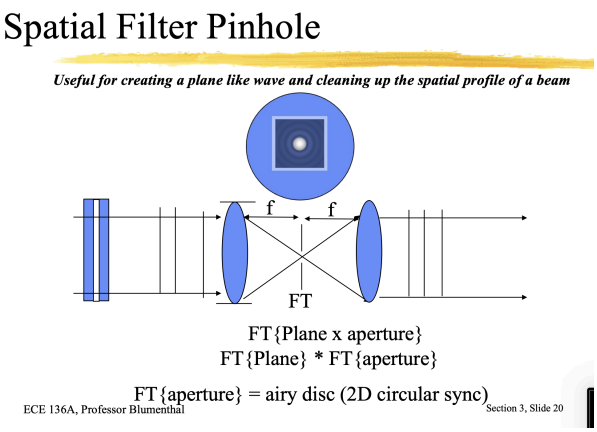
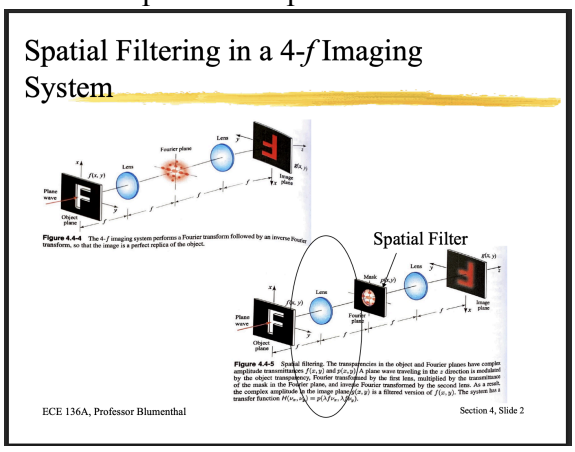


Figure 3: Spatial Filter Pinhole section 3, slide 20

The second objective explores the summation of spatial frequencies. Individual objects are decomposed to their specific spatial frequencies and reconstructed at the back focal plane of the lens. Structures such as gratings produce diffraction orders that are placed inversely proportional to the lattice constant. Placing a variable slit in the Fourier plane, can filter specific spatial frequencies. Additionally, the lattice constant of the grating determines the diffraction pattern's separation



.

The final objective of the lab builds on spatial filtering theory using contrasting masks. In a 4-f system, where the back focal plane is where the fourier transform lies. This plane holds the diffraction patterns in the form of bright spots. Each spot corresponds to a different frequency component. By selectively blocking or removing diffraction orders, the frequency spectrum can be suppressed. Blocking the zeroth order terms removes the low frequency components, improving fine details in the object.

This final objective through contrasting lens explores Babinet's principle. Babinet's principle uses complementary apertures to produce identical patterns. However, although the patterns are identical the intensity is inverted. This principle shows that diffraction depends on mask geometry. In fourier optics, an aperture and its inverse carry identical spatial frequencies.

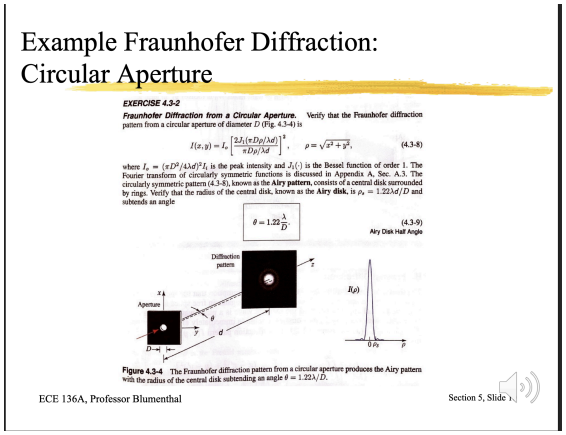
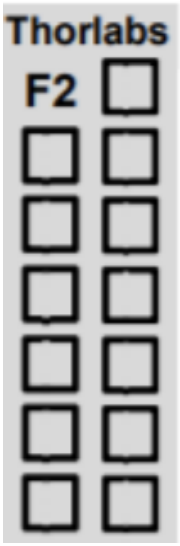


Figure 4: Spatial Filter Pinhole section 5, slide 10

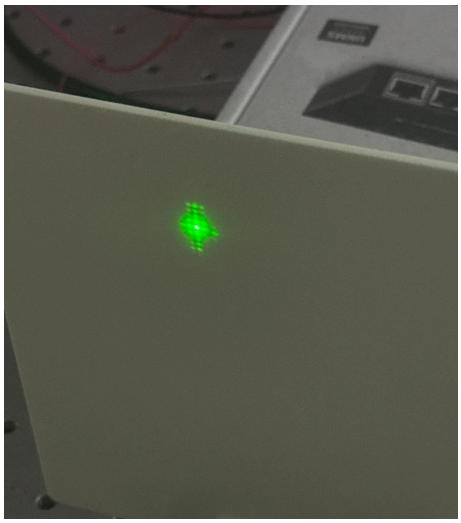
3 Experimental Procedure and Data

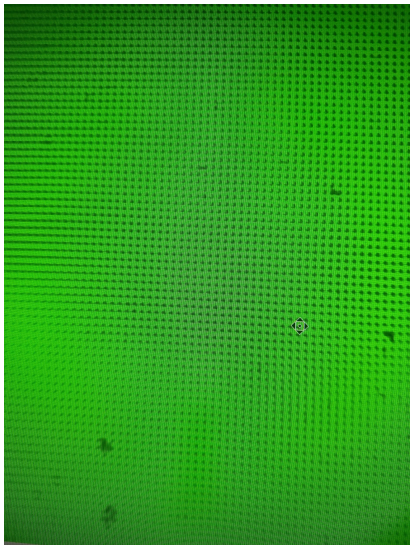
3.1 Fourier Image and Aperture Effects

Procedure:
The optical system used in the lab is a 4-f imaging system. It consists of an objective lens with a focal length of 5cm and a projection lens with focal length of 7.5cm. A beam splitter is used to capture the image and fourier plane in conjunction. The first mask was the Field F2 with the square lattice with the lattice constant of g=15 um and bar width b = 6um. The lattice image was placed into the focus on the camera and the resulting fourier plane. The fourier plane was located at the back of the objective lens and projected through a 4-f system introducing a beam splitter to project the fourier plane onto a screen and the image plane of the grating onto a camera.



The observed fourier image on the screen was a bright central point surrounded by weaker spots. When the projection lens got closer to the beam splitter, the image size increased. The fourier plane was always located on the back focal length of the objective lens. The sharpest observable fourier image was observed at an object distance of 2f+3 (f=7.5cm) and image distance of 2f where (f=7.5cm).





We saw the image in the beam path with a Keplerian telescope orientation, ultimately capturing the image with a camera.

Finally, opening and closing the aperture iris affected the size and intensity of the spots on the spots in the fourier plane captured by the screen on the back focal length of the projection lens. A smaller diameter resulted in larger, dimmer spots and a larger diameter resulted in smaller, sharper spots. When the field iris was manipulated the fineness of the object remained and instead the fourier pattern itself grew smaller and bigger.

3.2 Slit Filtering and Grating Fourier Transforms

Procedure:

Continuing from the same 4-f set up (objective lens, f= 5cm, and projector lens, f=7.5cm) from part 1, a slit filter is introduced in the fourier plane to manipulate frequency filtering on the fourier plane.

The experiment starts with the F2 lattice in the object plane which results in a diffraction pattern in the Fourier plane.

The slit is then adjusted to manipulate light transmission in the following ways:

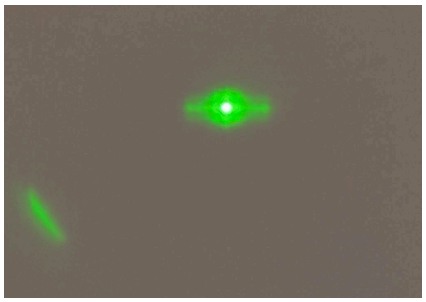
1. 0° (horizontal slit)
2. 90° (vertical slit)
3. 45° (diagonal slit)

At each corresponding slit angle, the resulting fourier plane is projected on the screen through the 4-f system as well as the reconstructed image through the camera is recorded.

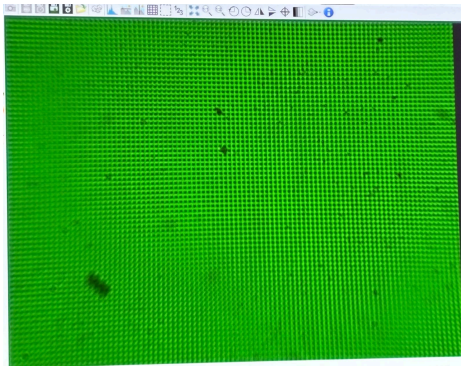
This procedure is repeated for different lattices: F2, F4, F6, F10, F12: Each with its unique lattice constants and varying spatial frequencies.

The filtered and unfiltered images for each field are recorded.

F2 Grating Data



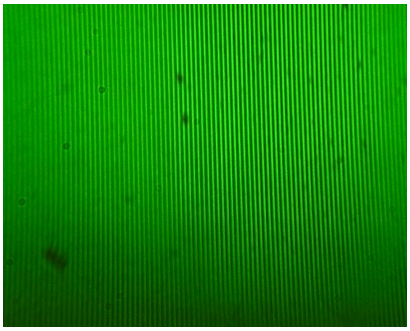
Unfiltered Fourier Plane



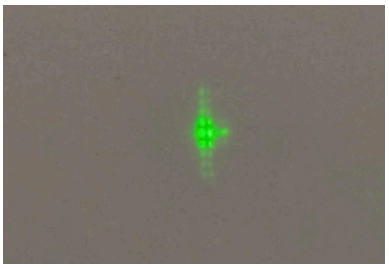
Unfiltered Image Plane



Filtered Fourier Plane - 0° (horizontal slit)



Filtered Image Plane - 0° (horizontal slit)



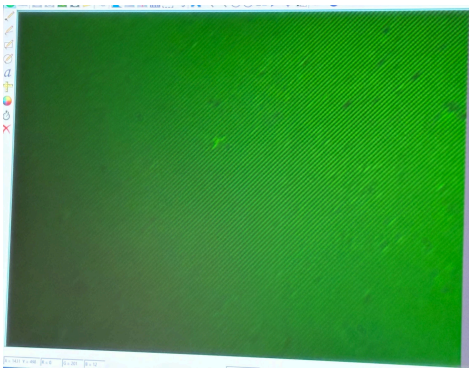
Filtered Fourier Plane - 90° (vertical slit)



Filtered Image Plane - 90° (horizontal slit)



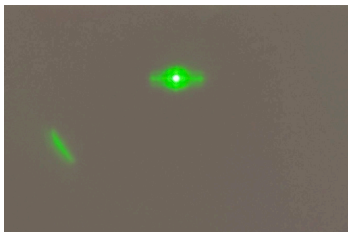
Filtered Fourier Plane - 45° (diagonal slit)



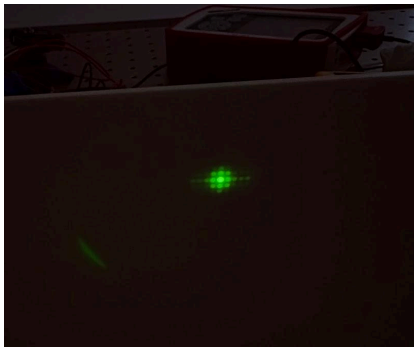
Filtered Image Plane - 45° (horizontal slit)

Lattice constant comparison (F4 vs F2)

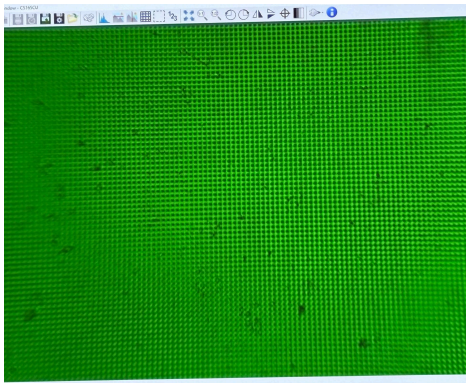
The F2 fourier and image planes were compared to the F4 grating fourier and image planes. It was noticed that the spacing between grids was tighter in F2 compared to F4. In contrast, the fourier image of F2 had larger spacing in the fourier plane concentrated to the tighter spacing of F4 in the fourier plane.



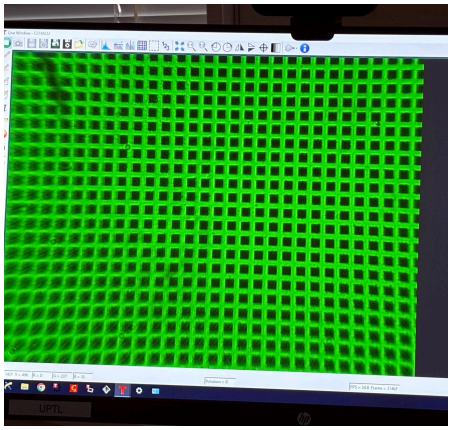
Unfiltered Fourier Plane - F2



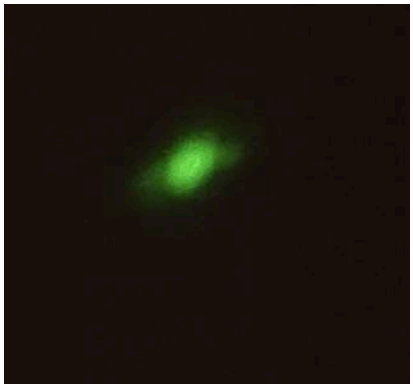
Unfiltered Fourier Plane - F4



Unfiltered Image Plane - F2

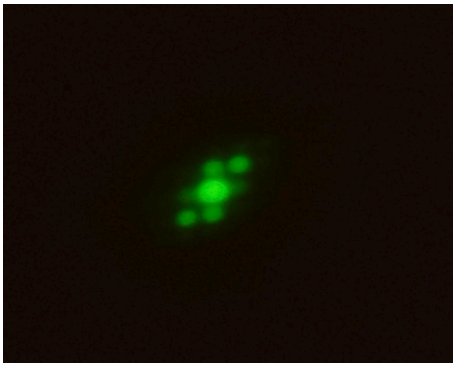


Unfiltered Image Plane - F4

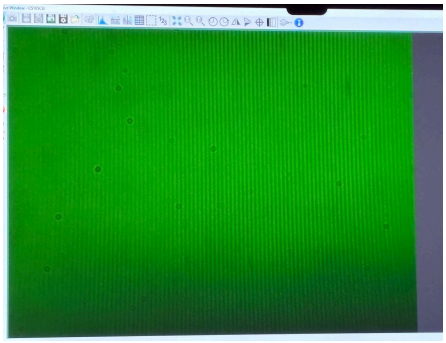


Filtered Fourier Plane - 0° (horizontal slit)

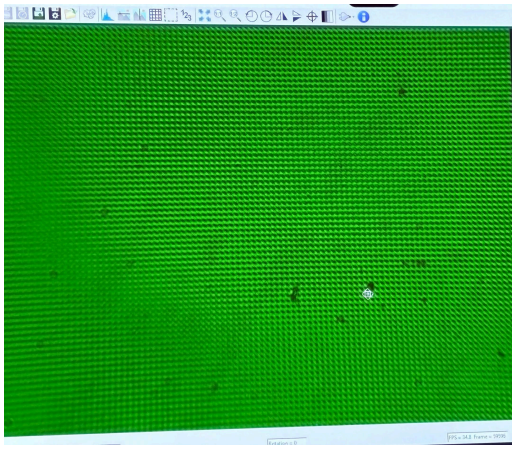
F6 Grating Data



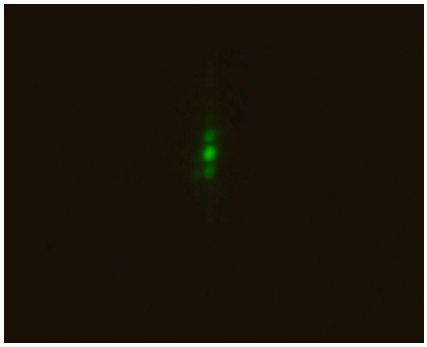
Unfiltered Fourier Plane



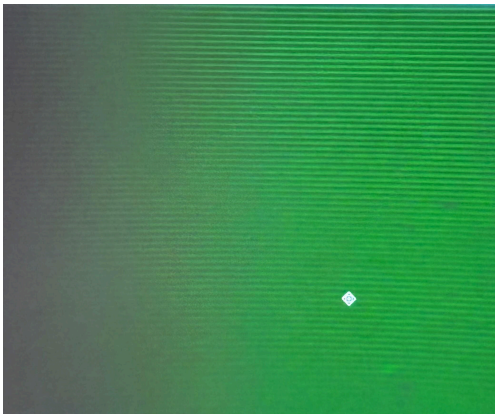
Filtered Image Plane - 0° (horizontal slit)



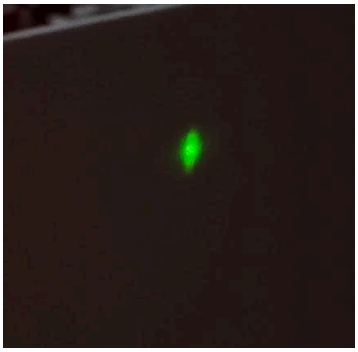
Unfiltered Image Plane



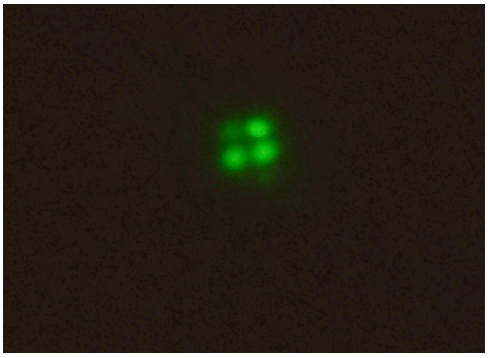
Filtered Fourier Plane - 90° (vertical slit)



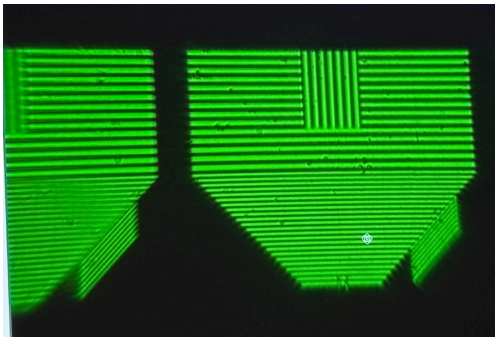
Filtered Image Plane - 90° (horizontal slit)



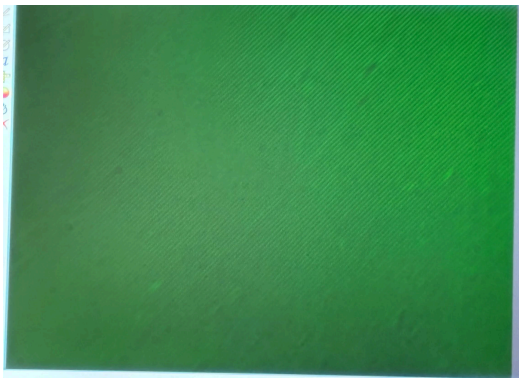
Unfiltered Fourier Plane



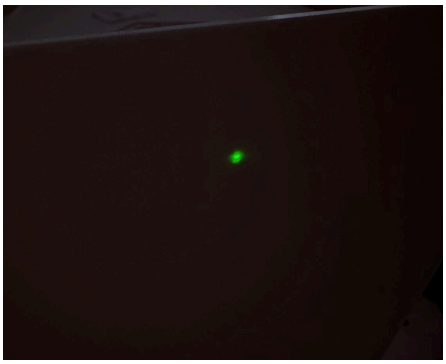
Filtered Fourier Plane - 45° (diagonal slit)



Unfiltered Image Plane

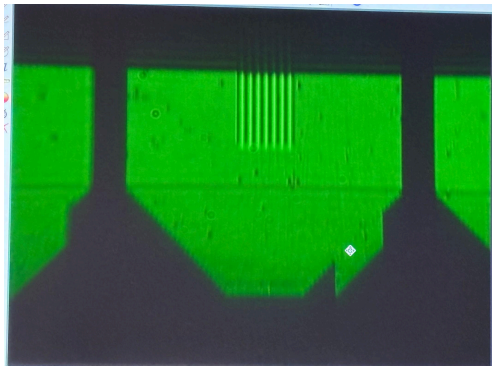


Filtered Image Plane - 45° (horizontal slit)

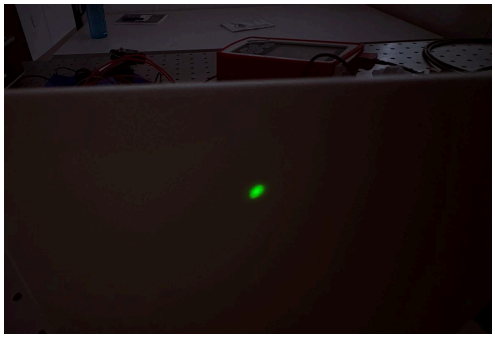


Filtered Fourier Plane - 0° (horizontal slit)

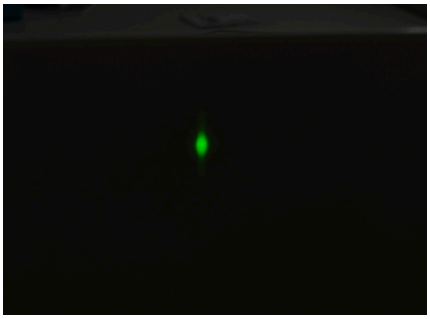
F10 Grating Data



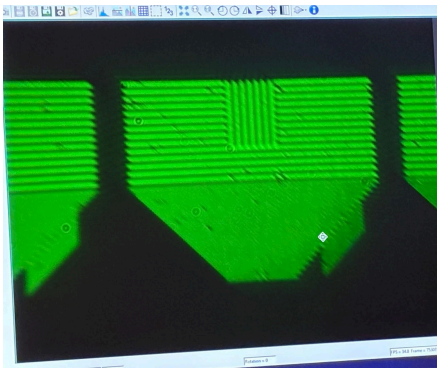
Filtered Image Plane - 0° (horizontal slit)



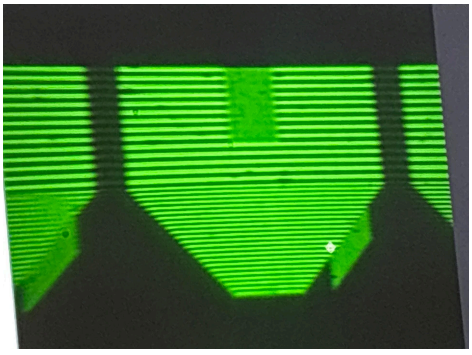
Filtered Fourier Plane - 45° (diagonal slit)



Filtered Fourier Plane - 90° (vertical slit)

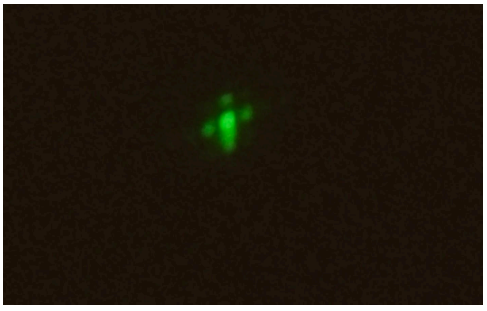


Filtered Image Plane - 45° (horizontal slit)

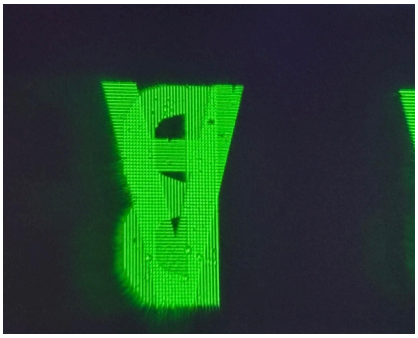


Filtered Image Plane - 90° (horizontal slit)

F12 Grating Data



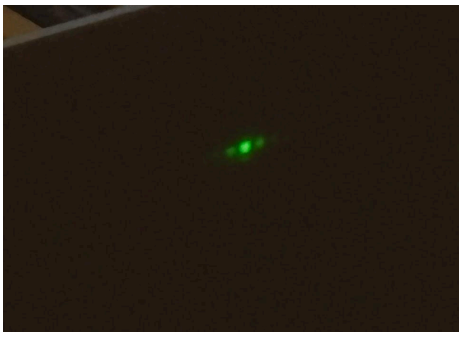
Unfiltered Fourier Plane



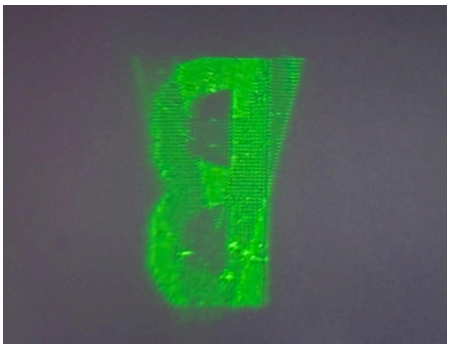
Unfiltered Image Plane



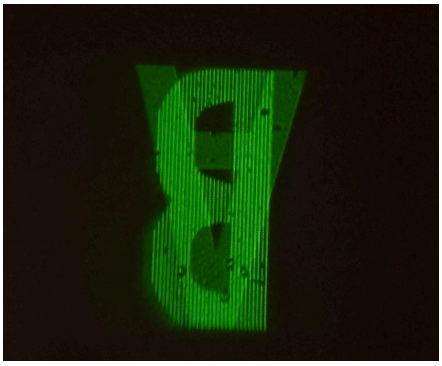
Filtered Fourier Plane - 90° (vertical slit)



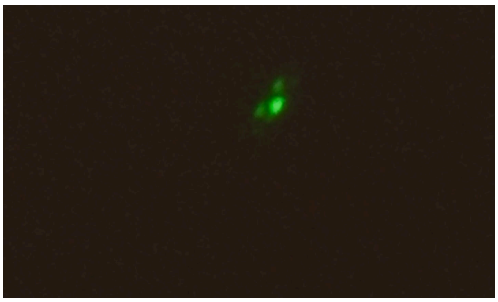
Filtered Fourier Plane - 0° (horizontal slit)



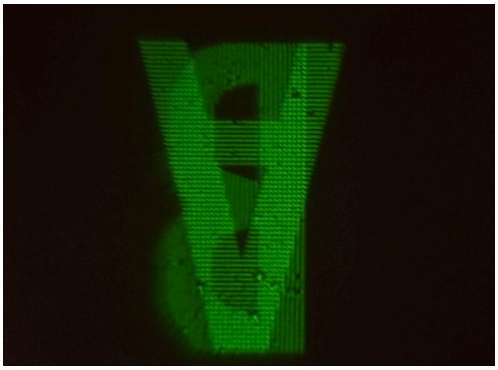
Filtered Image Plane - 90° (horizontal slit)



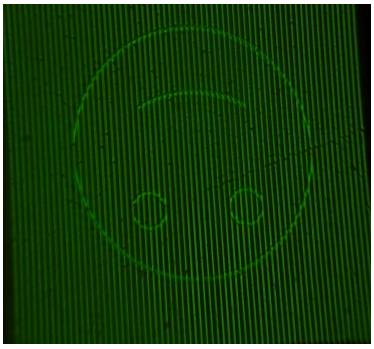
Filtered Image Plane - 0° (horizontal slit)



Filtered Fourier Plane - 45° (diagonal slit)

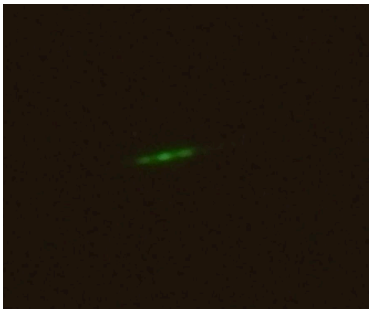


Filtered Image Plane - 45 (horizontal slit)



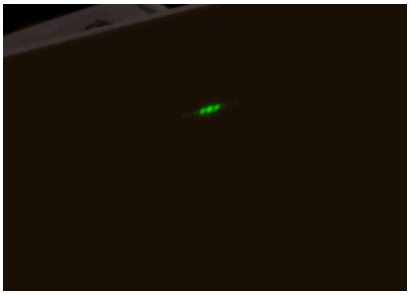
Unfiltered Image Plane

The target was replaced with the F9 pattern, a smiley face behind bars. The slit, positioned in the Fourier plane, was adjusted to free the smiley behind the vertical bars. Then, the slit was adjusted to allow only first and second order diffraction components to get through.

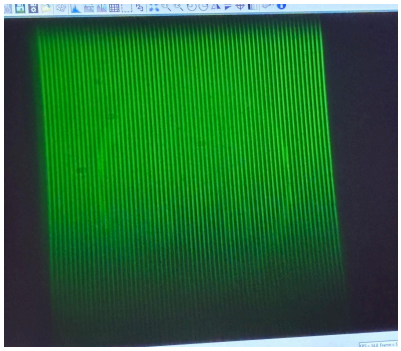


Filtered Fourier Plane - 0° (horizontal slit)

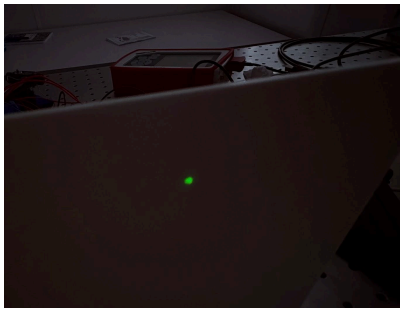
F9 Grating Data



Unfiltered Fourier Plane



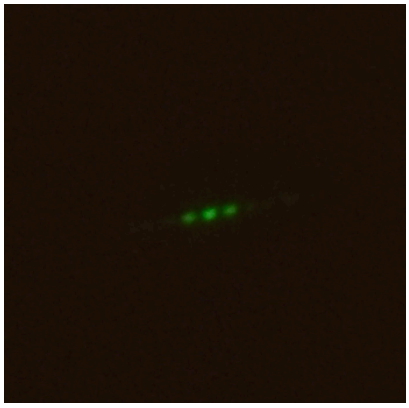
Filtered Image Plane - 0° (horizontal slit)



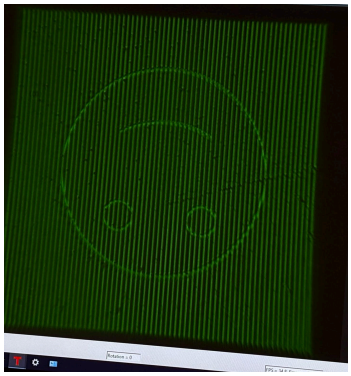
Filtered Fourier Plane - 90° (vertical slit)



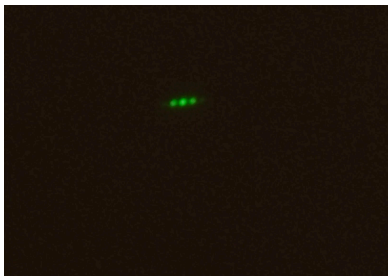
Filtered Image Plane - 90° (horizontal slit)



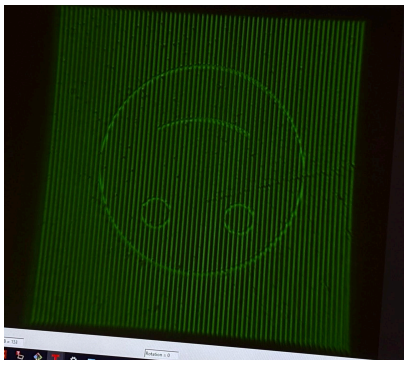
First Order fourier plane



First Order image plane



Second Order fourier plane



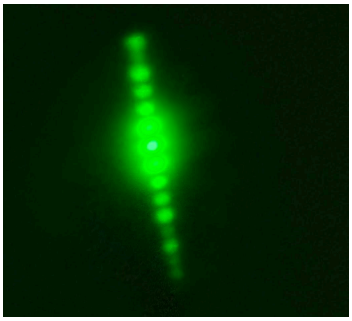
Second Order image plane

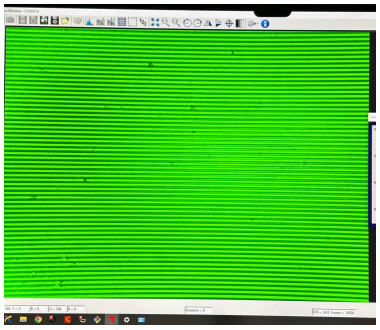
3.3 Fourier Image Masking

Keeping the same layout from parts 1 and 2. Instead of keeping a variable slit at the fourier plane, we will mount a EDU-TGC1 cutting mask at the fourier plane. We will center the mask spot on the zeroth order. We will see the

fourier plane and image plane to notice the fourier pattern is sharp.

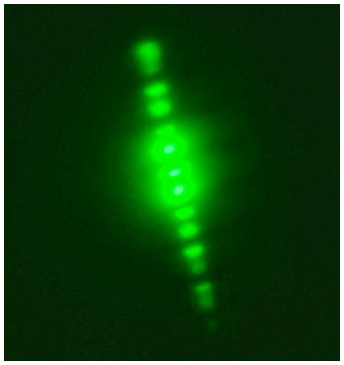
First we will switch the backlight target to a F8 line lattice and capture the unfiltered image plane and fourier plane.

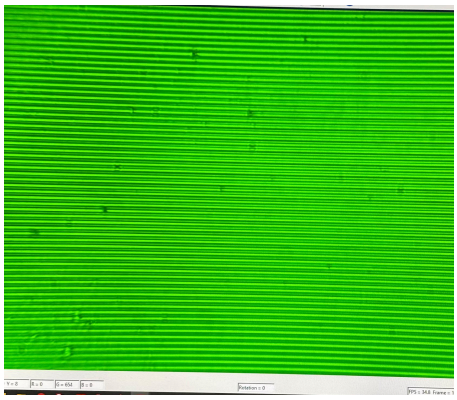




Before the zeroth order filtering mask filter.

After applying mask 1 and the zeroth order is filtered.





After the fourier space filtering.

Then, position the new target, triangles (f5), and capture the unfiltered image and fourier plane. Apply to mask 2 to block alternating diffraction maximums along three diffraction lines in the fourier plane. It was not possible to mask every other maximum, but a noticeable triangular pattern was observed after mask 2 was applied.

Next switch to the F13 Babinet field. Record the baseline with no mask with the fourier plane and image plane. Then, place a circular spot on the mask to block the zeroth order.

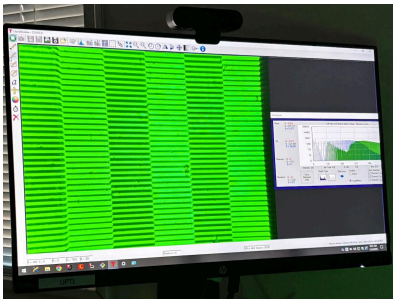
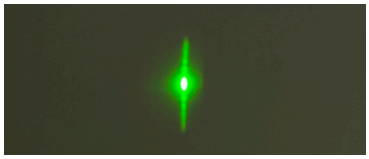


Image plane unfiltered



Fourier plane unfiltered

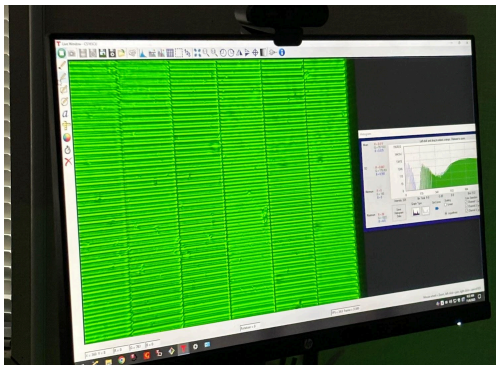
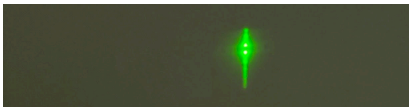
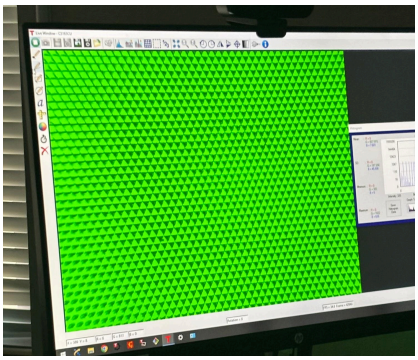


Image plane filtered zeroth order

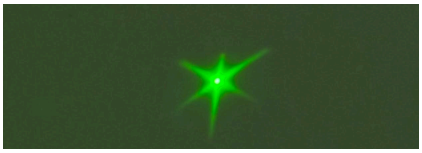


Fourier plane after filtering

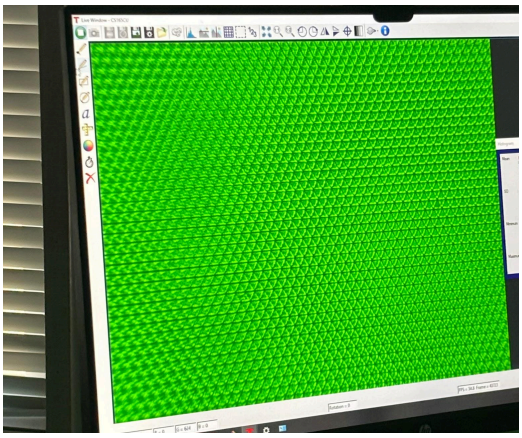
Repeat the same procedure on F5 the triangle mask. Capture the unfiltered image of the fourier and image plane. Then apply the mask and block out the zeroth order.



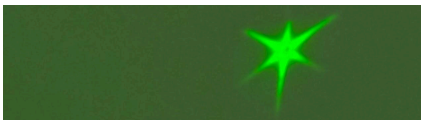
Triangle mask image unfiltered



Triangle Fourier image unfiltered

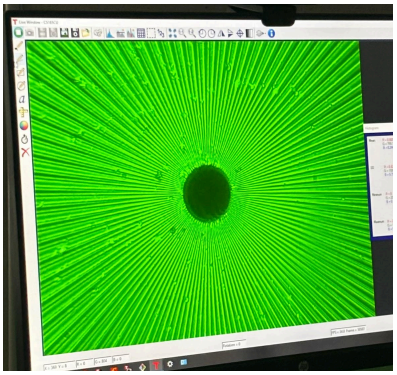


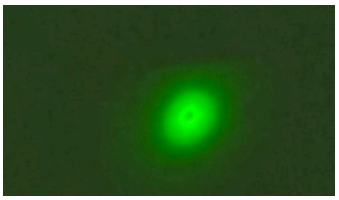
Triangle mask zeroth order image filtered



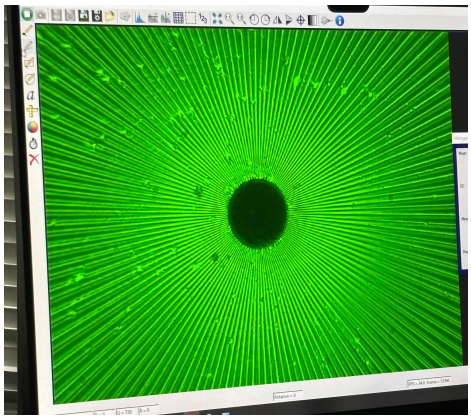
Triangle mask zeroth order fourier filtered

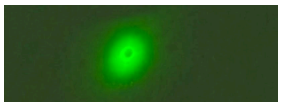
Switch the to the sector star masking center and apply in order in increasing radii the circular masks at the fourier plane and record the image along with the fourier plane



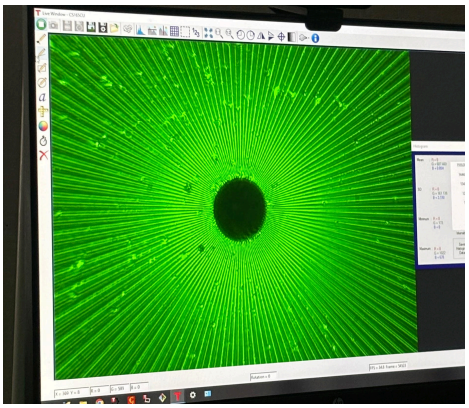


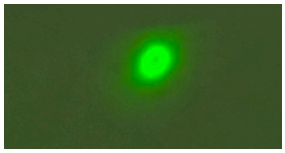
First smaller radii mask





medium radii mask





largest radii mask

4 Discussion, Analysis

4.1 Fourier Image and Aperture Effects

Part 1 - Fourier Image of Grating (F2)

When the square lattice (F2) was back light and focused through a 4-f system the resulting fourier plane displayed a bright spot surrounded by evenly spaced spots. These spots were 2x2 and displayed vertically and horizontally. When the fourier transform of a 2d pulse train is taken the resulting spatial transformation is a 2d since function. This is evident in the results of the fourier plane. Each spot corresponds to a spatial frequency component of the lattice. The spacing between spots is the inverse of the lattice length. If the grating was switching to 1-d the resulting fourier image observed would be a 1-d sinc function distribution and appear as a line instead of a grid of diffraction dots.

Part 2- Sharp image of the Target and Predicted Location

The predicted image before recording the observable image was 2f in the object plane and 2f in the back plane of the projection lens. This is because the thin lens equation shows us that an object at distance 2f will roughly translate through the lens and return an image with equal magnification.

During the lab, a sharp image was resolved when the screen was roughly 2f at the back focal length of the projection lens. The projection lens has a focal length of 7.5cm. The object plane was located roughly 2f+3 or 18 cm in the front plane of the projection lens. The resulting magnification calculated was around 15/18 ~ 0.83 which is near the magnification of the initial object. There were degrees of error present in our measurement because it was hard to measure the distances associated with the beam splitter. These errors were mitigated by keeping consistent measurement techniques near the center of each lens and optical component. This position gave us the most resolvable images and obeyed the thin lens equation as when the screen was moved closer it was operating between the f-2f region and appearing more magnified at the cost of resolution.

These focal distances verified that the Fourier plane could be seen on the screen through a 4-f system that projected the fourier plane on the back of the objective lens through a beam splitter and projection lens.

Part 3- Optical Elements effects on Diffraction Spots

Controlling the diameter of the aperture iris changed the size and intensity of the observable diffraction spots on the screen, capturing the

Fourier plane. A smaller aperture results in larger spots that are dimmer. A larger aperture resulted in smaller spots that were sharper in resolution. The field iris limited the illuminated area of the diverging source beam, resulting in either shrinking or expanding in the fourier plane. The resolution of the structures remained intact.

These adjustments showed us that the aperture iris was responsible for diffraction spot size and resolution on the fourier image displayed on the screen; whereas, the field iris controlled illuminated area and total image brightness following Kohler illumination theory.

4.2 Single Slit Filtering on Fourier Transform of Gratings

Part 1 - Filtering Gratings (F2, F4, F6, F10, F12)

For all the gratings common patterns emerged. The unfiltered Fourier plane displayed a bright central diffraction point surrounded by evenly spaced diffraction spots, corresponding to different frequencies.

When the slit was positioned at 0°, the horizontal slit, the illuminated object had only its horizontal frequencies pass through and the reconstructed image on the screen consisted of vertical lines.

When the slit was positioned at 90°, the vertical slit, the illuminated object had only its vertical frequencies pass through and the reconstructed image on the screen consisted of horizontal lines.

When the slit was positioned at 45°, both the vertical and horizontal frequencies contributed diagonally. The reconstructed object was oriented at 45° stripes on the axes.

Part 2 - Filtering of Smiley Behind Bars Pattern

When the Smiley face target was illuminated without filtering then the camera displaced a smiley face behind bars. Changing the slit vertically filtered the vertical bars and only let the Smiley frequencies through. Changing the slit horizontally resulted in the smiley face disappearing and only the bars remained.

When the width of the slit was reduced only the first order diffraction components got through and a faint version of the smiley appeared on the camera. Allowing only the second order components produced a dim smiley with lower resolution.

The results obeyed the theory that manipulating slit affected Fourier plane frequencies which changed the final images resolution and Fourier planes diffraction spot features.

The lattice constant determines the spacing between diffraction spots on the fourier plane. A smaller lattice constant produces larger separation between diffraction spots on fourier images. A larger lattice constant resulted in closer diffraction orders. This inverse relationship shows an inverse relation where the fourier image becomes more compact with higher lattice constants and vice versa.

The orientation of angle of the slit and width of slit changes the change in spatial frequencies. The slit’s angle changes the spatial frequency's direction which results in different lines on the final image. The slit width controls the frequency components which affects the resolution. Narrow slits act as a low pass filter that results in broad, smoother features. The wider slits let higher frequencies through and result in finer details. The final image directly relates to the filtered spatial frequencies.

The smiley remains visible because the diffraction order has the object’s spatial frequency. The first order shows a clearer image because it preserves central frequencies. The second order appears dimmer than the first

because there are more differing amplitudes and phases.

4.3 Fourier Image Masking

Part 1
When Mask 1 blocked every other diffraction maximum in the F8 line lattice, the resulting image plane had less repetition and was dimmer. The mask removed alternating frequency components which eliminated frequency information. This effectively doubled the spacing of the visible lines on the camera's real planar image.

Part 2

The results in the triangle grating proved hard to mask every other maximum. However, it was easy to notice that the pattern appeared tighter and was starting to lose symmetry. Mask 2 resulted in partial frequency suppression as seen in the change in detail in the image.

Part 3

When the zeroth order for F13 was blocked through the Babinet field, the image became dimmer and displayed high detail outlines. The zeroth order contains the DC intensity. Therefore, blocking it removes lower frequencies and enhances detail.

Part 4

Filtering the zeroth order for the triangle grating (F5) was similar because it made a darker version of the initial triangles. This proved that removing the central frequencies emphasized the higher frequencies.

Part 5

For the sector to start increasing the radius of the circular mask in the fourier plane dimmed the image. Additionally, the lower centered frequencies blocked caused the edges to dominate the image. Blocking larger portions in the center removed greater levels of low frequency information, leaving only the finer details.

Part 6

Babinet's principle shows that complementary contrasting masks produce symmetric diffraction patterns for the same overall intensity. This is shown through the experiment because masking and unmasking the same regions in the Fourier plan create contrasting images. One is dominated with brighter features and the other contrasting dark outlines. However, both have the same spatial content in their fourier planes.

Part 7

The sector star behavior is similar to the linear gratings. When the center is removed the lower frequencies disappear, increasing the fine details and reducing the brightness. The effective lattice constant is determined by spacing between the points of the stars. Both images show how higher frequencies are associated with finer details.

5 Conclusion

Through the experiments, the relationship between spatial frequencies and the fourier plan was established. In Part 1, varying the aperture affected the Fourier plane confirmed diffraction pattern theory. In Part 2, selectively filtering spatial frequencies at the fourier plane affected the focal plane and the final image. Finally, Part 3 showed how masking the Fourier plane controlled the final resulting image and affected image resolution. The lab reinforced image formation principles and spatial frequency manipulation through fourier optics. The spatial frequency affects resolution and structure of the final image and fourier plane.

References

1. Saleh, B. E. A., & Teich, M. C. *Fundamentals of Photonics*, 2nd ed., Wiley, 2007.

2. Lecture Slides, “Week 3–5” Prof. Blumenthal, UCSB ECE 136A, Fall 2025.
3. Thorlabs**.** *EDU-FOP2 Fourier Optics Educational Kit – Supplementary Manual, Rev. A.* Thorlabs, Inc., Newton, NJ.